Illinois Institute of Technology Homework 5

# Strength; Weakest Preconditions; Substitutions

*CS 536: Science of Programming; Due Wed Oct 5*

1. [4 points] Let *p*₀ → *p*₁, *p*₁ → *p*₂, *q*₀ → *q*₁, and *q*₁ → *q*₂ be valid. If {*p*₁} *S* {*q*₁} is valid, then which of the following triples are also valid [[1]](#footnote-1) ?
   1. {*p*₀} *S* {*q*₀}
   2. {*p*₀} *S* {*q*₂}
   3. {*p*₂} *S* {*q*₀}
   4. {*p*₂} *S* {*q*₂}

**Ans: b**

For Problems 2 – 4, syntactically calculate the *wp* but do not logically simplify the result. See the solution of Activity 8 for the level of detail to give.

1. [9 points] Calculate *wp*(**if** b[M] < v **then** L := M **else** R := M **fi**, *q*) where *q* ≡ b[L] < v ≤ b[R] ∧ L < R.

**Ans:** *B* ≡ b[M] < v

S ≡ **if** b[M] < v **then** L := M **else** R := M **fi**

w1 ≡ wp(L := M, b[L] < v ≤ b[R] ∧ L < R)

≡ b[M] < v ≤ b[R] ∧ M < R

w2 ≡ wp(R := M, b[L] < v ≤ b[R] ∧ L < R)

≡ b[L] < v ≤ b[M] ∧ L < M

Then wp(S,q) ≡ (*B →* w1) ∧ (¬*B →* w2)

≡ (b[M] < v *→* b[M] < v ≤ b[R] ∧ M < R) ∧ (b[M] ≥ v *→* b[L] < v ≤ b[M] ∧ L < M)

1. [7 points] Calculate *wp*(***if*** odd(x) ***then*** *S*₁; *S*₂ ***fi***, *q*) where *S*₁ ≡ x := x-y, *S*₂ ≡ y := y-z, and *q* ≡ 0 ≤ x < n ∧ y **=** z\*(n-x).

Ans: B = odd(x)

S=S1;S2

w2 ≡ wp(*S*2,q) ≡ 0 ≤ x < n ∧ y-z **=** z\*(n-x)

w1 ≡ wp(*S*₁, w2) ≡0 ≤ x-y < n ∧ y-z **=** z\*(n- x-y)

w3 ≡ wp(*skip*, q) ≡0 ≤ x < n ∧ y **=** z\*(n-x)

Then *wp*(***if*** odd(x) ***then*** *S else skip* ***fi***, *q*)

≡ (B→ w1) ∧ (¬*B →* w3)

≡ (odd(x)→ 0 ≤ x-y < n ∧ y-z **=** z\*(n- x-y)) ∧ (even(x) *→* 0 ≤ x < n ∧ y **=** z\*(n-x))

1. **[5 points] Calculate** *wp*(i := i-1; s := s+i, 0 ≤ i ≤ n ∧ s = sum(i,n)).

Ans: *wp*(i := i-1; s := s+i, 0 ≤ i ≤ n ∧ s = sum(i,n))

= *wp*(i := i-1, *wp*(s := s+i, 0 ≤ i ≤ n ∧ s = sum(i,n)))

= *wp*(i := i-1, 0 ≤ i ≤ n ∧ s+i = sum(i,n))

=0 ≤ i-1 ≤ n ∧ s+i-1 = sum(i-1,n))

For Problems 5 – 6, first calculate *wp*(the statement, postcondition). (a) What is the result? (b) Logically simplify your result from (a) to give a valid value for the precondition *p*.

1. [7 points] {*p*} j := i+j; i := i+1 {i ≤ j ∧ j-i < n}

Ans: wp(j := i+j; i := i+1, i ≤ j ∧ j-i < n)

= wp(j := i+j,wp(i := i+1, i ≤ j ∧ j-i < n))

= wp(j := i+j, i+1 ≤ j ∧ j- i+1 < n)

= i+1 ≤ i+j ∧ i+j - i+1 < n

b. (i+1 ≤ i+j) ∧ ((i+j - i+1) < n)

= (1≤ j) ∧ (j+1<n)

= (1≤ j) ∧ (j<n-1)

= 1≤ j< n-1

1. [7 points] {*p*} j := k-j; k := i+j {i ≤ j ≤ k}

Ans: wp(j := k-j; k := i+j, i ≤ j ≤ k)

= wp(j := k-j, wp(k := i+j, i ≤ j ≤ k))

= wp(j := k-j, i ≤ j ≤ i+j)

= i ≤ k-j ≤ i+ k-j adding j to all

= i+j ≤ k≤ i+ k substract I to all

= j ≤ k -i ≤ k

= j ≤ k ∧ i≥0

1. [15 points total] Let *p* ≡ x\*y < f(z) ∨ ∀x . x ≥ a → ∃y . x/y > y-a-z and calculate the following substitutions. Only do syntactic substitution, not algebraic simplification. For example, (x+2)[2/x] ≡ 2+2 is correct and is as far as you should go. Continuing via 2+2 = 4 is an algebraic simplification. Continuing via

2+2 ≡ 4 is wrong, since 2+2 ≢ 4.

* 1. [3 points] Calculate *p*[y-z/x].
  2. [3 points] Calculate *p*[y+z/y]
  3. [6 points] Calculate *p*[x+y/a][y\*z/x]
  4. [3 points] Calculate *p*[a/x][z/a]

Ans: a. *p* ≡ x\*y < f(z) ∨ ∀x . x ≥ a → ∃y . x/y > y-a-z

*p*[y-z/x] = (y-z)\*y < f(z) ∨ ∀x . x ≥ a → ∃y . x/y > y-a-z

b. *p* ≡ x\*y < f(z) ∨ ∀x . x ≥ a → ∃y . x/y > y-a-z

*p*[y+z/y]= x\* (y+z)  < f(z) ∨ ∀x . x ≥ a → ∃y . x/y > y-a-z

c. *p* = x\*y < f(z) ∨ ∀x . x ≥ a → ∃y . x/y > y-a-z

change bounded variable x and y to v and u respectively

p = x\*y < f(z) ∨ ∀v . v ≥ a → ∃u . v/u > u-a-z

*p*[x+y/a]=p1= x\*y < f(z) ∨ ∀v . v ≥  x+y  → ∃u . v/u > u-(x+y)-z

*p*[x+y/a][y\*z/x]

= p1[y\*z/x]= y\*z\*y < f(z) ∨ ∀v . v ≥  y\*z+y  → ∃u . v/u > u-(y\*z+y)-z

1. *p*[a/x]=p1= a\*y < f(z) ∨ ∀x . x ≥ a → ∃y . x/y > y-a-z

*p*[a/x][z/a]

*=*p1[z/a]= z\*y < f(z) ∨ ∀x . x ≥ z → ∃y . x/y > y-z-z

1. It doesn't matter if we assume validity under total or partial correctness — the answer is the same for both cases. [↑](#footnote-ref-1)